

SOLVING POLYNOMIAL INEQUALITIES

Objectives:

- 1) Solve polynomial inequalities algebraically (using a sign chart)
- 2) Solve polynomial inequalities using a graph

Solving polynomial inequalities hinge on finding the zeros. The zeros are called critical numbers. These numbers separate the number line into intervals and then the sign chart is used to determine the intervals in which the polynomial is either positive or negative.

Example 1: Solve $x^2 + 2x - 3 \geq 0$.

Factor to find the zeros

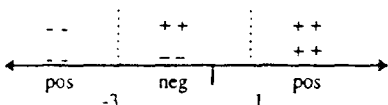
$$(x + 3)(x - 1) \geq 0$$

hence the zeros are -3 and 1.

Place the zeros -3 and 1 on the number line



Arbitrarily choose a value in each interval and determine whether + or -.



The solution includes the critical numbers because it has the \geq symbol, hence the solution is the intervals $(-\infty, -3]$ and $[1, \infty)$.

Example 2: Solve $x^3 < 9x$.

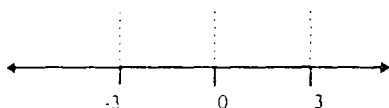
Factor to find the zeros

$$x^3 - 9x < 0$$

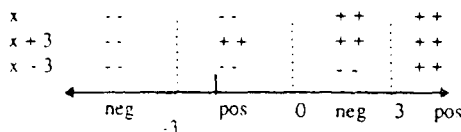
$$x(x + 3)(x - 3) < 0$$

hence the zeros are 0, -3, and 3.

Place the zeros -3, 0 and 3 on the number line



Arbitrarily choose a value in each interval and determine whether + or -.



The solution does **not include the critical numbers** because there is not equal symbol, hence the solution is the intervals $(-\infty, -3)$ and $(0, 3)$.

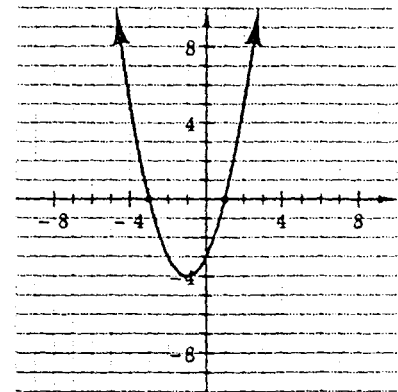
SOLVING USING THE GRAPH

Example 3: Solve $x^2 + 2x - 3 \geq 0$ using its graph.

The graph of $y = x^2 + 2x - 3$ is at the right.

The inequality asks for y values greater than or equal to zero, so the solution would be the interval(s) along the x-axis for which the graph is above the x-axis.

The solution is the intervals $(-\infty, -3]$ and $[1, \infty)$. It includes the values $f(x) = 0$, the x intercepts.

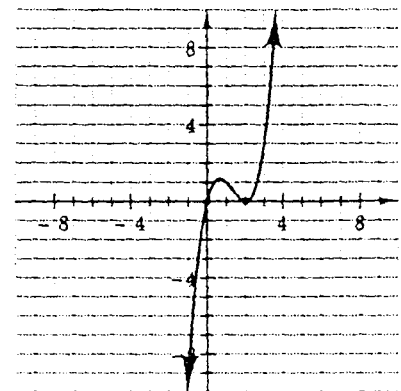


Example 4: Solve $x(x - 2)^2 < 0$.

The graph of $y = x(x - 2)^2$ is at the right.

The inequality asks for y values that are less than zero, so the solution would be the x interval for which the graph is below the x-axis.

The solution is the interval $(-\infty, 0)$. Why does the solution not include zero?



Polynomial Inequality Handout

Solve each of the following graphically. Write your answer in interval notation.

1. $x^2 - 2x \geq 0$

2. $25 - x^2 \geq 0$

3. $(x+3)^2 \leq 0$

4. $x(x-3)^2 \geq 0$

5. $x^2(4-x)(x+6) < 0$

6. $x^2 + 1 \leq 0$

Solve each of the following algebraically. Write your answer in interval notation.

7. $x^2 - 4x \geq 5$

8. $x^2 + 5 \leq 6x$

9. $x^4 < 4x^2$

10. $4x^2 > 12x - 9$

11. $2x^2 - 3 \leq x$

12. $7 - x^2 \leq 0$

13. $x^4 - 5x^2 \leq -4$

14. $x^5 + 9x \geq 10x^3$

15. $x^2 + 9 \leq 0$

16. $2x^3 - 5x^2 < 2x - 5$