

## NON-LINEAR SYSTEMS OF INEQUALITIES

Objectives: To graphically display the solution set for a non-linear system of equations.  
To incorporate all of the different base graphs learned during this course.

**Example 1:** Sketch the solution set to the system

$$\begin{cases} x^2 + y^2 \geq 16 \\ \frac{x^2}{9} + \frac{y^2}{49} \leq 1 \end{cases}$$

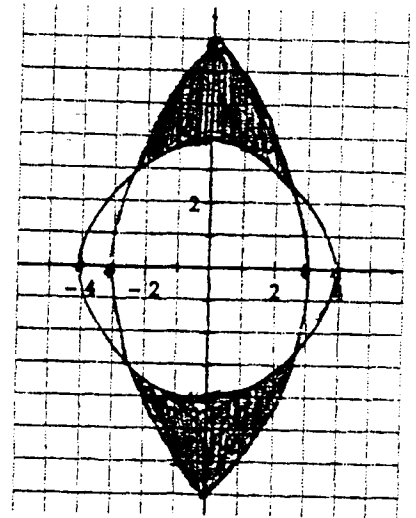
Graph each equation as an equality. The first is a circle whose radius is 4. Choose a point that is not on the circle, say  $(0, 0)$ , and determine if this point makes the inequality true or false. In this case  $0^2 + 0^2 \geq 16$  is not true so the region is outside the circle.

The second is an ellipse with vertices at  $(0, \pm 7)$ .

Choose a test point, say  $(0, 0)$ . Is  $\frac{0^2}{9} + \frac{0^2}{16} \leq 1$ ?

Since this is true the region is inside the ellipse.

The common area is outside the circle but inside the ellipse.

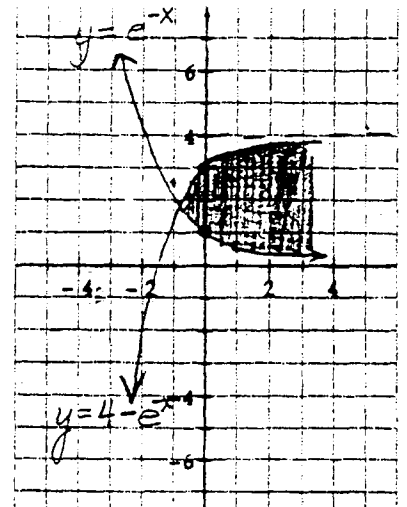


**Example 2:** Sketch the solution set to the system

$$\begin{cases} y \geq e^{-x} \\ y \leq 4 - e^{-x} \end{cases}$$

The first equation is the exponential base graph with the base less than one. Choose a test point, say  $(0, 0)$ . Is  $0 \geq e^0$ ? Since it is not, the region is above the base graph.

The second equation is the same base graph that has been reflected about the x-axis then moves four units up. Choose the test point  $(0, 0)$ . Is  $0 \leq 4 - e^0$ ? Since it is, the region is below the translated base graph.

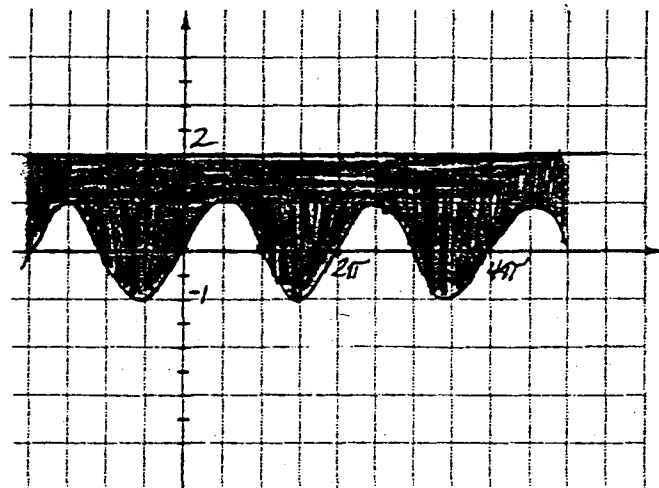


**Example 3:** Sketch the solution set to the system  $\begin{cases} y \geq \sin x \\ y \leq 2 \end{cases}$

The first equation is the sine base graph. If we choose the test point  $(0, 1)$ , notice that  $1 \geq \sin 0$  is true. Hence the region is above the base graph.

The second equation is a horizontal line. The region is below the line.

The common region is above the sine graph and below the horizontal line.



## Non-Linear Systems of Inequalities

Solve each of the following systems:

$$1. \begin{cases} x + 3y \leq 3 \\ 2x^2 + y^2 \geq 18 \end{cases}$$

$$2. \begin{cases} x^2 + y^2 < 13 \\ y \geq x^2 - 1 \end{cases}$$

$$3. \begin{cases} x^2 - y^2 \geq -5 \\ 3x^2 + 2y^2 \leq 30 \end{cases}$$

$$4. \begin{cases} x^2 + y^2 \geq 3 \\ x \geq y^2 \end{cases}$$

$$5. \begin{cases} y \leq 3^x + 2 \\ y \geq -1 \end{cases}$$

$$6. \begin{cases} y \geq \left(\frac{1}{4}\right)^x - 1 \\ y \leq 4 - x^2 \end{cases}$$

$$7. \begin{cases} x \leq 3 \\ y \geq \log_3 x \\ -2 \leq y \leq 5 \end{cases}$$

$$8. \begin{cases} y \geq \log_{1/2} x \\ y < 6 \\ x \leq 8 \end{cases}$$

$$9. \begin{cases} y \geq x \\ y \leq \cos x \\ x \geq 0 \end{cases}$$

$$10. \begin{cases} y \geq -x \\ y \leq 1 \\ y \geq \tan x \end{cases}$$

$$11. \begin{cases} y \geq -\sqrt{x+1} \\ y \leq x+1 \\ x \leq 3 \end{cases}$$

$$12. \begin{cases} \frac{x^2}{25} + \frac{(y-2)^2}{4} \leq 1 \\ y \geq \frac{1}{x^2} \end{cases}$$

$$13. \begin{cases} y \leq \ln(x+2) \\ y \geq |x| - 1 \end{cases}$$

$$14. \begin{cases} y \geq x^3 \\ x^2 - y^2 \leq 1 \end{cases}$$