FUNCTIONS

Objectives:

1. Definition of a function
2. Vertical line test
3. Domain and range
4. Even and odd functions

Algebra often attempts to determine the relationship between two variables. One of the variables is considered the **independent variable** and the other is the **dependent variable**. For example the cost to mail a package depends on its weight, hence the weight is the independent variable and the cost is the dependent variable. The notation used for this example would be:

C(w) read "C of w" where w is the weight and C(w) is the cost.

Another example is your electric bill depends on the number of kilowatt hours used during the month. E(n) where n is number of kilowatt hours and E(n) is the electric bill. Do you know which is the dependent variable? If you answer the electric bill, you are correct.

A. DEFINITION:

> A function is a process by which each value assigned to x, the independent variable, produces one and only one value for y, the dependent variable.

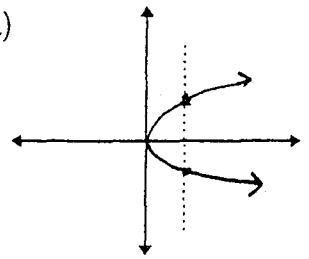
Example 1. Let y = 2x + 1. The function or process here is to double a number then add one. Arbitrarily let x = 3, then notice that twice three plus 1 is 7 (i.e. $2 \cdot 3 + 1 = 7$). Hence if x is 3 then the only value that is produced by the process for y is 7, so y is a function of x.

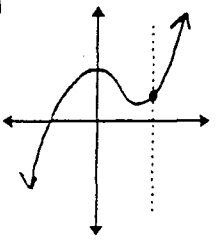
Example 2. Let $y = \pm\sqrt{x}$. Arbitrarily let x = 16, then $\pm\sqrt{16} = \pm 4$. Hence if x is 16 then there are two possible values for y, both -4 and 4, so y is <u>not</u> a function of x.

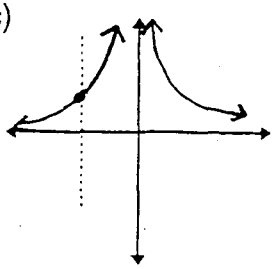
B. VERTICAL LINE TEST for FUNCTIONS:

> If a vertical line at any position intersects the graph in more than one point, the graph is **not** the graph of a function.

Example 3:

a) 

b) 

c) 

In this case example 3a) is not a function because the vertical line (dotted) crosses the curve in two locations. Examples 3b) and 3c) are functions since the vertical line **only** crosses the curve in **one** location.

C. DOMAIN & RANGE:

The domain is the set of values for the independent variable and the range is the set of values generated by the function.

Example 4: Find the domain and range of $f(x) = \sqrt{x}$.

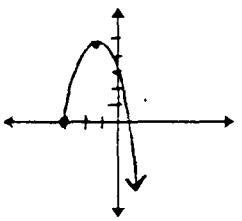
Since $f(x)$ contains a radical the values of x must be either positive or zero. Using interval notation, the domain is $[\,0, \infty)$.

The only values generated by the function are either zero or positive numbers. Using interval notation, the range is $[0, \infty)$.

Example 5: Find the domain of $f(x) = \dfrac{1}{x-5}$.

Since $f(x)$ contains a fraction any value that yields zero in the dominator must be excluded. The domain is the set of all real numbers except 5.

Example 6: State the domain and range of the graph, use interval notation.



The values for x begin at -3, hence the domain is $[-3, \infty)$.

The values for y begin at negative infinity and end at 5, hence the range is $(-\infty, 5\,]$.

E. EVEN & ODD FUNCTIONS:

A function is said to be **even** if its graph is symmetric with respect to the y-axis and to be **odd** if its graph is symmetric with respect to the origin.

Example 7: Name as many as possible of your base graphs that are even functions.

Possible answers are $f(x) = x^2$, $f(x) = |x|$, *and* $f(x) = \frac{1}{x^2}$.

Algebraic test for even and odd functions:
A function *f(x)* is even if, for each x in the domain of *f*, $f(-x) = f(x)$.
A function *f(x)* is odd if, for each x in the domain of *f*, $f(-x) = -f(x)$.

Example 8: The function $f(x) = 2x^3$ is odd because replacing all the x by -x yields $f(-x) = 2(-x)^3 = -2x^3 = -f(x)$.

Example 9: The function $f(x) = \frac{1}{x^2 - 4}$ is even because replacing all the x by -x yields $f(-x) = \frac{1}{(-x)^2 - 4} = \frac{1}{x^2 - 4} = f(x)$.

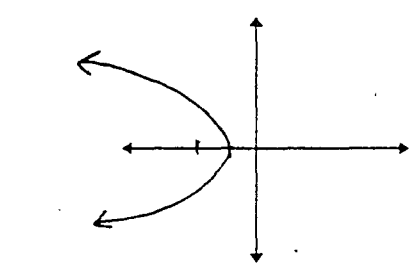
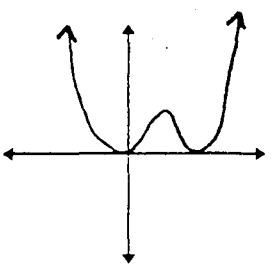
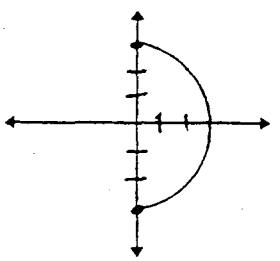
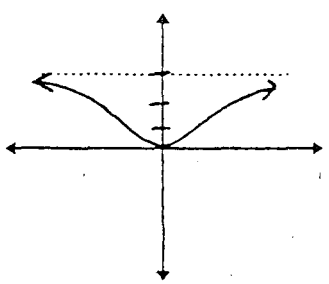
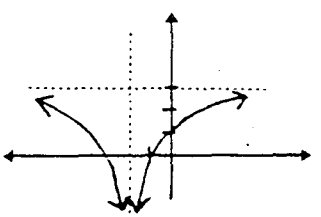
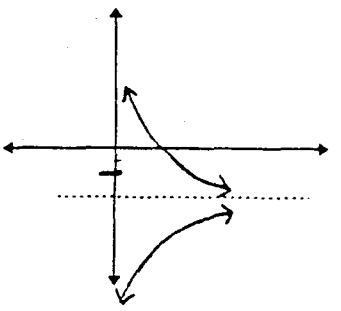
FUNCTION HANDOUT -- Homework assignment

For each of the following, determine if y is a function of x. If it is not, give **two specific points** which prove it is not a function.

1. $y = \frac{1}{2}x^2$
2. $2x = y^2$
3. $y = |x|$
4. $x = |y|$
5. $y = x^3$
6. $x = y^3$
7. $y = 1 - \frac{1}{4}x$
8. $x = \frac{4-y}{4}$
9. $x = 2$
10. $y = -3$
11. $y = \sqrt{9-x^2}$
12. $y = \frac{5}{x-6}$
13. $y = -\sqrt{x+3}$
14. $y = \frac{1}{(x+2)^2}$
15. $y = -\frac{1}{x} + 2$

16. Find the domain and range for problems 1,3,5,7,9,10,13,14, and 15. Use interval notation.

17. Which of the following graphs represent functions? Name the domain and range for each using interval notation.

QUADRATIC FUNCTIONS

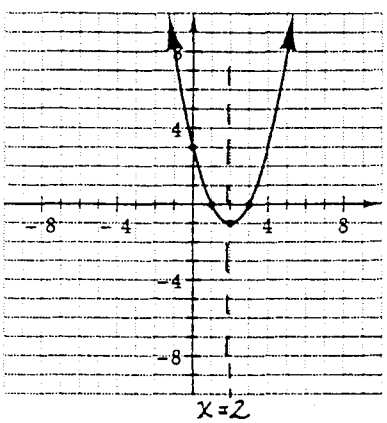
Objectives:

1) definition of a quadratic function
2) find the x and y intercepts
3) find the vertex using complete the square
4) naming the axis of symmetry

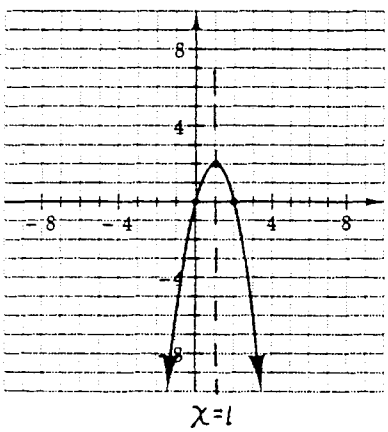
DEFINITION -- A function of the form $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers with $a \neq 0$, is called a **quadratic function**. The graph of such a function is called a **parabola**.

The important properties of a quadratic function are its intercepts, vertex, and axis of symmetry. See the two examples below. The intercepts, vertex and axis of symmetry are marked on each graph.

$f(x) = x^2 - 4x + 3$



$f(x) = 4x - 2x^2$



A. THE INTERCEPTS:

To find the y-intercept, set x = 0, then solve for y.
To find the x-intercept, set $f(x) = 0$, then solve for x.

Example 1: Let $f(x) = x^2 - 4x + 3$, clearly state in coordinate form the intercepts. Also name the equation of the axis of symmetry.

To find the y-intercept, set x = 0.

$y = 0^2 - 4(0) + 3 = 3$,

hence the **y-intercept is (0, 3).**

To find the x-intercept, set $f(x) = 0$.

$$0 = x^2 - 4x + 3$$
$$0 = (x - 3)(x - 1)$$
$$x = 1,\ 3$$

Hence the **x-intercepts are (1, 0) and (3, 0).**

The axis (or line) of symmetry is such that if the curve were folded over on this line, its left half would coincide with its right half. So the line of symmetry is halfway between the two x-intercepts, that is, the line **x = 2 is the axis of symmetry**.

B. THE VERTEX:

To find the coordinates of the vertex, it is necessary for the function to be in the form $f(x) = a(x - h)^2 + k$ where the **vertex is (*h*, *k*)** and "*a*" determines the shape and direction of the parabola. In order to put the quadratic function in this form it is necessary to complete the square.

Example 2: Let $f(x) = x^2 - 4x + 3$, find the coordinates of the vertex then use the intercepts and the vertex to sketch the graph..

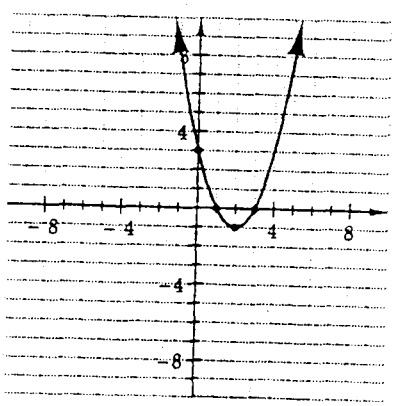
$f(x) = \underline{x^2 - 4x + \quad} + 3 -$ Complete the square by

adding then subtracting 4

$f(x) = \underline{x^2 - 4x + 4} + 3 - 4$ Factor the trinomial

$f(x) = (x - 2)^2 - 1$

Hence the **vertex is (2, -1).**



Example 3: Determine the vertex, x and y intercepts, and axis of symmetry for the graph of $f(x) = -x^2 + 3x + 10$.

Setting x = 0, leads to y = 10, hence the **y-intercept is (0, 10).**

Setting $f(x) = 0$, leads to $0 = -x^2 + 3x + 10$

$$0 = -\left(x^2 - 3x - 10\right)$$

$$0 = -(x-5)(x+2)$$

$$x = 5, -2$$

hence the **x-intercepts are (5, 0) and (-2, 0).**

To find the vertex, complete the square: $f(x) = -\left(x^2 - 3x\right) + 10$

$$f(x) = -\left(x^2 - 3x + \frac{9}{4}\right) + 10 + \frac{9}{4}$$

$$f(x) = -\left(x - \frac{3}{2}\right)^2 + \frac{49}{4},$$

hence the vertex is $\left(\frac{3}{2}, \frac{49}{4}\right)$.

The axis of symmetry goes through the vertex, so the equation is $x = \frac{3}{2}$.

Notice that "a" is -1 so the direction of the parabola is downward. **The vertex of a parabola is also called its maximum or minimum point.**

C. SHORT CUT FORM TO FIND THE VERTEX:

If the quadratic is in the form $f(x) = ax^2 + bx$, where the constant term is missing, or if the x-intercepts are easy to find then the easiest way to find the x coordinate of the vertex is to find the x-intercepts. The vertex is always located at their midpoint. Substitute the midpoint into the function will then yield the y coordinate of the vertex.

Example 4: Find the vertex and graph $f(x) = x^2 + 8x$.

By factoring the x-intercepts are 0 and -8. Their midpoint is -4. Therefore the x coordinate of the vertex is -4. Now substitute -4 into the function.

$$y = (-4)^2 + 8(-4) = 16 - 32 = -16.$$

The vertex is (- 4, - 16)

QUADRATIC FUNCTIONS - homework assignment

In exercise 1-8 graph the quadratic function defined by the equation. On the graph indicate

a) the coordinates of the x and y intercepts
b) the equation of the axis of symmetry
c) the coordinates of the vertex
d) the range of the function

1. $f(x) = x^2 + 2x - 24$
2. $f(x) = -x^2 - 4x + 5$
3. $f(x) = 2x^2 + 8x$
4. $f(x) = x^2 - 4$
5. $f(x) = 4x^2 - 4x - 3$
6. $f(x) = x^2 - 5x$
7. $f(x) = x^2 + 10x + 25$
8. $f(x) = -x^2 - 4$

In exercise 9-12 determine the vertex of each function by completing the square.

9. $f(x) = x^2 - 2x + 3$
10. $f(x) = x^2 + 6x - 1$
11. $f(x) = -x^2 + x + 2$
12. $f(x) = 2x^2 + 4x - 5$

13. What is the domain for all the functions in exercises 1-10?

14. Find two positive numbers whose sum is 20 and whose product is a maximum.

15. The height of a ball thrown directly up from a roof 144 ft high with an initial velocity of 128 ft/sec is given by the formula $y = 144 + 128t - 16t^2$. What is the maximum height attained by the ball?

16. The sum of the base and altitude of a triangle is 12 inches. Find the base and altitude if the area is to be a maximum.