

FACTORING

Objective:

1. Factor greatest common factor
2. Factor trinomials
3. Factor by grouping
4. Factor difference of perfect squares
5. Factor sum and difference of cubes
6. Factor with fraction and negative exponents
7. Factor with variable exponents

The first five types listed above are treated as review. It is assumed that you have been exposed to many of these types and are accurate at factoring them. The intention here is to combine them into multi-step problems and include trigonometry. The last two types are treated as new topics.

Review examples:

Factor each completely.

a) $y^4 - 2y^3 - 9y^2 + 18y$ 1) factor the common "y"
 $= y(y^3 - 2y^2 - 9y + 18)$ 2) factor by grouping
 $= y[y^2(y - 2) - 9(y - 2)]$ 3) factor the common (y - 2)
 $= y(y - 2)(y^2 - 9)$ 4) factor the difference of squares
 $= y(y - 2)(y - 3)(y + 3)$

b) $\sin^3 x + 27$ 1) factor the sum of cubes
 $= (\sin x + 3)(\sin^2 x - 3\sin x + 9)$

c) $18 \cos^2 x \tan x - 57 \cos x \tan x + 24 \tan x$ 1) factor common "3tan x"
 $= 3 \tan x (6 \cos^2 x - 19 \cos x + 8)$ 2) factor the trinomial
 $= 3 \tan x (2 \cos x - 1)(3 \cos x - 8)$

FACTORIZING WITH FRACTIONAL OR NEGATIVE EXPONENTS:

Rule: Identify any base(s) common to all the terms. The greatest common factor is the factor with the smallest exponent.

Example 1: For each of the following identify the common factor(s) and the GCF.

a) $x^{-3} + 2x^{-1}$ The common factor is "x" and the GCF is x^{-3} .

b) $x^{-2}y^2 - x^2y^{-1}$ The common factors are "x" and "y" and the GCF is $x^{-2}y^{-1}$.

c) $4x^{-1}y^{-\frac{1}{2}} + 2xy^{\frac{1}{2}}$ The common factors are "2", "x" and "y" and the GCF is $2x^{-1}y^{-\frac{1}{2}}$.

Example 2: Factor each completely:

a) $5x^{-1} + 3x$ The GCF is x^{-1}
hence $5x^{-1} + 3x$
= $x^{-1}(5 + 3x^2)$

b) $x^{\frac{1}{2}}(x+2)^{-3} + 3x^{-\frac{1}{2}}(x+2)^{-2}$ The GCF is $x^{-\frac{1}{2}}(x+2)^{-3}$

hence $x^{\frac{1}{2}}(x+2)^{-3} + 3x^{-\frac{1}{2}}(x+2)^{-2}$
= $x^{-\frac{1}{2}}(x+2)^{-3} [x + 3(x+2)]$
= $x^{-\frac{1}{2}}(x+2)^{-3} [4x + 6]$
= $2x^{-\frac{1}{2}}(x+2)^{-3} [2x + 3]$

FACTORING - Homework Assignment

Factor each completely.

1. $4x^3 - 8x^2 - 9x + 18$

2. $(2x + 5)^2 + 9(2x + 5)$

3. $25x^2 - 40xy + 16y^2$

4. $4\cos^2 x - 13\cos x + 3$

5. $(x - y)^2 - 4$

6. $(\tan x - 1)^2 + 3(\tan x - 1)$

7. $\sin^2 x + 2\sin x \cos x + \cos^2 x - 9$

8. $\tan^3 x + 1$

9. $8x^3 - 125$

10. $(x - y)^3 + 27$

11. $x^{2/3} - 5x^{1/3} - 24$

12. $x - 5x^{1/2} + 6$

13. $(x + 3)^{-2} - 3(x + 3)^{-1} - 10$

14. $x^2 - a^2 + x - a$

15. $(x + 2)^{-1}(x - 1)^{-3} - (x + 2)^{-2}(x - 1)^{-2}$

16. $2x^{-3/2} - 8x^{1/2}$

17. $(y + 2)^{1/5} - (y + 2)^{-4/5}$

18. $x^{1/2}(x - 3)^{-1/2} + x^{-1/2}(x - 3)^{-3/2}$

19. $x^{n+1} + 3x^n$

20. $x^{2n} + 6x^n + 9$

21. $y^{4n} - 81$

22. $x^{2n+2} + x^{n+2} - 30x^2$

23. $6x^{n+2} - 3x^{n+1} - x^n$