

Algebra ReviewI. Complex Fractions

Recall that a complex fraction is one in which there are fractions in the numerator, denominator, or both. The fastest way to simplify a complex fraction is to multiply the numerator and denominator by the least common denominator of all of the fractions which appear inside.

Example 1

To simplify the complex fraction  $\frac{3 - \frac{1}{x^2}}{2 - \frac{3}{x}}$  we would multiply the numerator and denominator by the LCD of the fractions which is  $x^2$ :

$$\frac{\left(3 - \frac{1}{x^2}\right)x^2}{\left(2 - \frac{3}{x}\right)x^2} = \frac{3x^2 - 1}{2x^2 - 3x}$$

Example 2

To simplify the complex fraction  $\frac{\frac{b}{b-a} - \frac{a}{b+a}}{\frac{b^2+a^2}{b^2-a^2}}$  we would multiply the numerator and denominator by the LCD of the fractions which is  $b^2 - a^2 = (b-a)(b+a)$ :

$$\begin{aligned} \frac{\left(\frac{b}{b-a} - \frac{a}{b+a}\right)(b-a)(b+a)}{\left(\frac{b^2+a^2}{b^2-a^2}\right)(b-a)(b+a)} &= \frac{b(b+a) - a(b-a)}{b^2+a^2} \\ &= \frac{b^2 + ab - ab + a^2}{b^2+a^2} = \frac{b^2+a^2}{b^2+a^2} = 1 \end{aligned}$$

## II. Negative and Fractional Exponents

To simplify expressions with negative or fractional exponents, we use the same properties of exponents with which you are already familiar as well as the following property:  $a^{-n} = \frac{1}{a^n}$ . Expressions with fractional exponents can also be written in radical form but we will be studying that later in the course. For now, we will simply be using the properties of exponents.

### Example 1

$$2^{1/2} \cdot 2^{3/2} = 2^{1/2 + 3/2} = 2^{4/2} = 2^2 = 4$$

### Example 2

$$\frac{5^{1/3}}{5^{4/3}} = 5^{1/3 - 4/3} = 5^{-3/3} = 5^{-1} = \frac{1}{5}$$

### Example 3

$$\left(a^{-2} b^6\right)^{-1/2} = \left(a^{-2}\right)^{-1/2} \left(b^6\right)^{-1/2} = a^1 b^{-3} = a \cdot \frac{1}{b^3} = \frac{a}{b^3}$$

### Example 4

$$(x+y)^{5/3} (x+y)^{-2/3} = (x+y)^{5/3 + -2/3} = (x+y)^{3/3} = x+y$$

### Example 5

$$\begin{aligned} \left(\frac{b^{1/3}}{b^{4/3} + b^{7/3}}\right)^{-1} &= \frac{1}{\frac{b^{1/3}}{b^{4/3} + b^{7/3}}} = \frac{b^{4/3} + b^{7/3}}{b^{1/3}} = \frac{b^{4/3}}{b^{1/3}} + \frac{b^{7/3}}{b^{1/3}} \\ &= b^{4/3 - 1/3} + b^{7/3 - 1/3} = b^{3/3} + b^{6/3} = b + b^2 \end{aligned}$$

ALGEBRA REVIEW  
HOMEWORK ASSIGNMENT

I. COMPLEX FRACTIONS

Simplify each of the following complex fractions:

1. 
$$\frac{z + 4 - \frac{5}{z}}{z + 1 - \frac{2}{z}}$$

2. 
$$\frac{1 + \frac{3}{x+1}}{\frac{4}{x^2 - 1}}$$

3. 
$$\frac{\frac{1}{\sin\theta + 1} + \frac{1}{\sin\theta - 1}}{\frac{1}{\sin\theta - 1} - \frac{1}{\sin\theta + 1}}$$

4. 
$$\frac{\frac{3}{(x+h)^2} - \frac{3}{x^2}}{h}$$

5. 
$$\frac{\frac{x}{1+x} - \frac{1-x}{x}}{\frac{x-1}{x} - \frac{x}{x+1}}$$

6. 
$$\frac{1 - \frac{\sin x}{\cos x}}{\cos x - \frac{\sin^2 x}{\cos x}}$$

7. 
$$\frac{1 + \frac{1}{n}}{\frac{1}{n+1} - 1}$$

8. 
$$\frac{\frac{2^{n+1}}{n 3^n}}{\frac{2^n}{(n-1) 3^{n-1}}}$$

## II. FRACTIONAL AND NEGATIVE EXPONENTS

Simplify each of the following expressions. Write your final answer without negative exponents.

$$9. \frac{xy^{-1} + x^{-1}y}{x^{-1}y - xy^{-1}}$$

$$10. (x^{-1} + y^{-1})^{-1}$$

$$11. \frac{x^{-1/4}}{x^{1/2}}$$

$$12. \frac{y+3}{(y+3)^{-1/2}}$$

$$13. \left( \frac{x^{1/4} y^{-2/3} z^0}{x^{3/4} y^{-2/3} z^{1/2}} \right)^{-3/2}$$

$$14. (x^{1/2} - x^{-1/2})^2$$

$$15. \frac{x^{1/3} + x^{-2/3}}{x^{1/3} - x^{-2/3}}$$

$$16. (a^{x^2+2x} \cdot a)^{\frac{1}{x+1}}$$